

Pensieve Header: The degree growth rate?

```

In[53]:=  $\beta$ Simplify = (# // Together // ExpandDenominator // ExpandNumerator) &;
SetAttributes[ $\beta$ Collect, Listable];
 $\beta$ Collect[B[ $\omega$ _,  $\mu$ _]] := B[
   $\beta$ Simplify[ $\omega$ ],
  Collect[ $\mu$ , _h, Collect[#, _t,  $\beta$ Simplify] &]
];
(* "L" for "Labels" *)
hL[ $\beta$ _] := Union[Cases[ $\beta$ , h[s_]  $\Rightarrow$  s, Infinity]];
tL[ $\beta$ _] := Union[Cases[ $\beta$ , t[s_] | Ts  $\Rightarrow$  s, Infinity]];
dL[ $\beta$ _] := Union[hL[ $\beta$ ], tL[ $\beta$ ]];
SetAttributes[ $\beta$ Form, Listable];
 $\beta$ Form[B[ $\omega$ _,  $\mu$ _]] := Module[
  {tails, heads, mat},
  tails = tL[B[ $\omega$ ,  $\mu$ ]]; heads = hL[B[ $\omega$ ,  $\mu$ ]];
  mat = Outer[ $\beta$ Simplify[Coefficient[ $\mu$ , h[#1] t[#2]]] &, heads, tails];
  PrependTo[mat, t /@ tails];
  mat = Prepend[Transpose[mat], Prepend[h /@ heads,  $\omega$ ]];
  MatrixForm[mat]
];

In[9]:= R[x_, y_] := B[1, (Tx - 1) t[x] h[y]];
Rinv[x_, y_] := B[1, (1 / Tx - 1) t[x] h[y]];
tm[x_, y_, z_][ $\beta$ _] :=  $\beta$  /. {t[x]  $\rightarrow$  t[z], t[y]  $\rightarrow$  t[z], Tx  $\rightarrow$  Tz, Ty  $\rightarrow$  Tz};
hm[x_, y_, z_][B[ $\omega$ _,  $\mu$ _]] := Module[
  { $\gamma$ x = D[ $\mu$ , h[x]],  $\gamma$ y = D[ $\mu$ , h[y]], M =  $\mu$  /. h[x] | h[y]  $\rightarrow$  0},
  B[ $\omega$ , M + h[z] ( $\gamma$ x +  $\gamma$ y + ( $\gamma$ x /. t[i_]  $\Rightarrow$  1)  $\gamma$ y)] //  $\beta$ Collect
];
swap[x_, y_][B[ $\omega$ _,  $\mu$ _]] := Module[
  { $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ ,  $\epsilon$ },
   $\alpha$  = Coefficient[ $\mu$ , h[x] t[y]];
   $\beta$  = D[ $\mu$ , t[y]] /. h[x]  $\rightarrow$  0;
   $\gamma$  = D[ $\mu$ , h[x]] /. t[y]  $\rightarrow$  0;
   $\delta$  =  $\mu$  /. h[x] | t[y]  $\rightarrow$  0;
   $\epsilon$  = 1 +  $\alpha$ ;
  B[ $\omega$  *  $\epsilon$ , Plus[
     $\alpha$  (1 + ( $\gamma$  /. t[i_]  $\Rightarrow$  1) /  $\epsilon$ ) h[x] t[y],
     $\beta$  (1 + ( $\gamma$  /. t[i_]  $\Rightarrow$  1) /  $\epsilon$ ) t[y],
     $\gamma$  /  $\epsilon$  h[x],
     $\delta$  - (1 /  $\epsilon$ )  $\gamma$  *  $\beta$ 
  ]] //  $\beta$ Collect
];
gm[x_, y_, z_][ $\beta$ _] :=  $\beta$  // swap[y, x] // hm[x, y, z] // tm[x, y, z];
B /: B[ $\omega$ 1_,  $\mu$ 1_] B[ $\omega$ 2_,  $\mu$ 2_] := B[ $\omega$ 1 *  $\omega$ 2,  $\mu$ 1 +  $\mu$ 2];

```

```
In[63]:= n = 5;
Flatten[{
   $\beta$ Form[ $\beta = B[1, \text{Sum}[g \text{ Random}[] t[i] h[j], \{i, n\}, \{j, n\}]]],$ 
  Table[
    {
       $\beta$ Form[ $\beta = \beta // \text{gm}[1, k, 1],$ 
      Cases[ $\beta, g^k \_ \rightarrow k, \text{Infinity}] // \text{Max}$ 
    },
    {k, 2, n}
  ]
}] // ColumnForm
```

Out[64]=

	1	h[1]	h[2]	h[3]	h[4]	h[5]
t[1]	0.954821 g	0.593199 g	0.848112 g	0.625379 g	0.0197342 g	
t[2]	0.167297 g	0.105424 g	0.988806 g	0.507697 g	0.0710087 g	
t[3]	0.477353 g	0.493155 g	0.0413349 g	0.99735 g	0.305348 g	
t[4]	0.777411 g	0.380916 g	0.280423 g	0.289995 g	0.517866 g	
t[5]	0.313304 g	0.172979 g	0.250118 g	0.0706627 g	0.358483 g	

1 + 0.593199 g	h[1]	h[3]	h[4]
t[1]	$\frac{1.82074 g + 5.6604 g^2 + 5.50282 g^3 + 1.65251 g^4}{1. + 1.1864 g + 0.351885 g^2}$	$\frac{1.83692 g + 1.97767 g^2}{1. + 0.593199 g}$	$\frac{1.13308 g + 1.32694 g^2}{1. + 0.593199 g}$ $\frac{0.09074}{1.}$
t[3]	$\frac{0.970507 g + 1.71467 g^2 + 0.675635 g^3}{1. + 1.1864 g + 0.351885 g^2}$	$\frac{0.0413349 g - 0.39373 g^2}{1. + 0.593199 g}$	$\frac{0.99735 g + 0.283219 g^2}{1. + 0.593199 g}$ $\frac{0.305}{1.}$
t[4]	$\frac{1.15833 g + 1.8093 g^2 + 0.66568 g^3}{1. + 1.1864 g + 0.351885 g^2}$	$\frac{0.280423 g - 0.156713 g^2}{1. + 0.593199 g}$	$\frac{0.289995 g - 0.0661916 g^2}{1. + 0.593199 g}$ $\frac{0.517}{1.}$
t[5]	$\frac{0.486283 g + 0.774495 g^2 + 0.288314 g^3}{1. + 1.1864 g + 0.351885 g^2}$	$\frac{0.250118 g + 0.00166434 g^2}{1. + 0.593199 g}$	$\frac{0.0706627 g - 0.0662601 g^2}{1. + 0.593199 g}$ $\frac{0.3584}{1.}$

1. + 2.43012 g + 1.97767 g ²	h
t[1]	$\frac{4.6695 g + 74.0449 g^2 + 553.648 g^3 + 2583.28 g^4 + 8394.37 g^5 + 20084.5 g^6 + 36484.4 g^7 + 51143.2 g^8 + 55}{1. + 12.6865 g + 75.6932 g^2 + 279.924 g^3 + 713.232 g^4 + 1317.8 g^5 + 1810.74 g^6 + 1867}$
t[4]	$\frac{1.43875 g + 9.02982 g^2 + 24.354 g^3 + 35.4}{1. + 6.04663 g + 15.9789 g^2 + 23.021 g}$
t[5]	$\frac{0.7364 g + 4.69825 g^2 + 12.8265 g^3 + 18.8}{1. + 6.04663 g + 15.9789 g^2 + 23.021 g}$

$\frac{1. + 7.58386 g + 25.3096 g^2 + 46.284 g^3 + 48.235 g^4 + 26.6742 g^5 + 5.89075 g^6}{1. + 3.02332 g + 3.41921 g^2 + 1.17315 g^3}$	t[1]	$\frac{8.52867 g + 450.575 g^2 + 11713.7 g^3 + 199639. g^4 + 2.50738 \times 10^6 g^5}{1. + 49.0112 g + 1180.45 g}$
	t[5]	

$\frac{1. + 16.6749 g + 131.59 g^2 + 648.245 g^3 + 2218.68 g^4 + 5563.2 g^5 + 10508.3 g^6 + 15148.1 g^7 + 16700.9 g^8 + 13975.3 g^9 + 8712.4 g^{10} + 3912.21 g^{11} + 11}{1. + 11.2004 g + 57.9493 g^2 + 180.55 g^3 + 372.527 g^4 + 528.683 g^5 + 519.578 g^6 + 346.978 g^7 + 149.669 g^8 + 37.4217 g^9 + 4.}$	t[1]
---	------